Heat Transfer

Turbulent Flow Heat Transfer

Analyzing heat transfer with turbulent flow, we are stuck in a bad place analytically. We cannot begin to solve the equations of motion for an isothermal flow, much less for a flow with changing temperature. To develop the necessary engineering relations between the Nusselt's number, Reynolds' number and other measurable characteristics of turbulent flow, we use analogies to give us the relevant parameters to describe the situation:

Reynolds stated two mechanisms contribute to the transport of heat:

1) Internal diffusion of the fluid when at rest
2) Eddies caused by bulk motion which continually brings fresh fluid into contact with the surface.

\[ H = At + B\rho v t \]

\( t = \) temperature difference  
\( \rho = \) density \hspace{1cm}  
\( v = \) bulk velocity \hspace{1cm}  
\( A, B = \) constants  
\( H = \) heat transmitted / area-time

An analogous equation was written for fluid friction

\[ R = A'v + B' \rho v^2 \]

Reynold's intuition led to the belief

\[ A \propto A' \hspace{1cm} B \propto B' \hspace{1cm} \text{or} \hspace{1cm} h \propto f \]

Consider bulk flow through a pipe at fluid temperature \( t_b \) and pipe temperature \( t_s \), the four quantities are:

\begin{itemize}
  \item[a)] heat flux to the pipe wall: \( h(t_b - t_s) \)
  \item[b)] momentum flux at wall: \( \tau_s g_c \)
  \item[c)] bulk energy flow: \( w C_p (t_b - t_s) \)
  \item[d)] bulk momentum flow: \( wu_b \)
\end{itemize}
It was then postulated that
\[ \frac{a}{c} = \frac{b}{d} \]
or that the ratio of the flux to the amount of heat/or momentum is equal.

This means also that
\[ h = \tau_{g_c} \frac{C_p}{u_b} \]

We also know that
\[ t_s g_c = \frac{f u_b^2 \rho}{2} \]
so
\[ h = \frac{f u_b \rho C_p}{2} \]
this expression is known as the Reynold's analogy.

Useful forms of the equations:
\[ h = \frac{f u_b \rho C_p}{2} \]

In terms of dimensionless numbers
\[ Nu = \frac{f}{2} Re Pr \]

For pipe flow
\[ f = 0.046 Re^{-0.2} \quad \text{or} \]
\[ Nu = 0.023 Re^{0.8} Pr \]

For shorter tubes, experimental correlations show that
\[ Nu = 1.86 Re^{1/3} Pr^{1/3} (L/D)^{-1/3} (\mu_b/\mu_D)^{14} \]

Fluid properties are evaluated at mean bulk fluid temperature
\[ \frac{(T_{b_{in}} + T_{b_{out}})}{2} \]

\( \eta_0 \) — viscosity at mean wall temperature
Turbulent Flow

$$\text{Nu} = 0.15 \text{Re}^{0.32} \text{Pr}^{0.14} \frac{(\mu_f)}{\mu_0}$$

for $\text{Re} > 10^4$ — Seider and Tate again

Properties are at average bulk temperature.

Dittus and Bolter $\text{Nu} = 0.023 \text{Re}^{0.8} \text{Pr}^{0.3}$ or $0.4$

$0.3$ — cooling $0.4$ — heating

Flow past a sphere

$$\text{Nu} = 2 + (0.4 \text{Re}^{1/2} + 0.06 \text{Re}^{2/3}) \text{Pr}^{0.4} \frac{(\mu_{\infty})^{1/4}}{\mu_0}$$

or cylinders

$$\text{Nu} = 2 + (0.4 \text{Re}^{1/2} + 0.06 \text{Re}^{6/7}) \text{Pr}^{0.4} \frac{(\mu_{\infty})^{1/4}}{\mu_0}$$

$\mu_{\infty}$ — free stream viscosity

$\mu_0$ — wall viscosity

Many heat transfer correlations

$$\text{Nu} = \alpha \text{Re} \text{Pr}^y \left(\frac{D}{L}\right)^b (\text{Gr})^w \left(\frac{\mu_f}{\mu_s}\right)^v$$

Of course, in all these situations, all of these coefficients aren't equally important.

Seider and Tate equation for laminar flow

$$\text{Nu}_m = 1.86 \text{Re}^{1/3} \text{Pr}^{1/3} \left(\frac{D}{L}\right)^{1/3} \left(\frac{\mu_f}{\mu_s}\right)^{0.14}$$

Since three of the coefficients have the same exponents, they can be combined

$$(\text{Re} \text{ Pr} \frac{D}{L})^{1/3} = \text{Graetz number}$$

Turbulent flow
Since the fluid is mixed, variations in viscosity are small and for $L/D >> 1$, the mixing removes any dependence downstream.

Dittus and Boelter Correlation

$$\text{Nu} = 0.023 \ (\text{Re})^{0.8} \ \text{Pr}^{0.3} \ or \ 0.4$$

properties at bulk temperature

Colburn equation

$$\frac{h}{u_b \ \rho \ C_p} \ (\text{Pr})^{2/3} = 0.023 \ \text{Re}^{-0.2}$$

properties at arithmetic mean of wall and bulk fluid temperatures

next approximation

add viscosity correction

$$\frac{h}{u_b \ \rho \ C_p} \ (\text{Pr})^{2/3} \left(\frac{\mu}{H_s}\right)^{0.14} = 0.23 \ \text{Re}^{-0.2}$$

Natural convection

$$\text{Nu} = 0.53 \ \text{Gr}^{1/4} \ \text{Pr}^{1/4}$$

For natural convection from horizontal cylinder (pipes)

$$\text{Nu} = 0.53 \ \left(\frac{d^3 \ \rho^{2/3} \ g \ \beta \ \Delta T}{\mu^2} \right)^{1/4} \ \text{Pr}^{1/4}$$

$\beta$ — coefficient of thermal expansion $- \frac{1}{p} \ (\frac{\partial p}{\partial T})_p$

$\Delta T$ — temperature difference from wall to $\infty$
Example:

A pipe contains steam such that the outer surface of the insulation is 50°C. The outside diameter of the insulation is .1 m and the room is at 20°C. Find \( h \) for the pipe.

1) Take average temperature for fluid properties
   \[
   (20 + 56)/2 = 38°C
   \]
   \[
   K = .0266 \text{ w/m} \cdot \text{k}
   \]
   \[
   \rho = 1.14 \text{ kg/m}^3
   \]
   \[
   \beta = .00322/k =
   \]
   \[
   \mu = 1.92 \times 10^{-5} \text{ Pa \cdot S}
   \]
   \[
   C_p = 1000 \text{ J/kg-k}
   \]
   \[
   \Delta T = 36°C
   \]
   \[
   Gr = 4.0 \times 10^6
   \]
   \[
   Pr = .72
   \]
   \[
   \frac{hD}{k} = \text{Nu} = (.53)(4.0 \times 10^6)^{1/4}(.72)^{1/4}
   \]
   \[
   h = 5.8 \text{ w/m}^2 \cdot \text{k}
   \]
   for natural convection from a vertical flat plate.

   \[
   \text{Nu} = .13 \text{ Gr}^{1/3} \text{ Pr}^{1/3} \text{ for turbulent flow}
   \]

Simplified versions of these reduce to

For \( GrPr > 10^9 \) \( h = .18 (\Delta T)^{1/3} \) all surfaces

   cylinders \( h \) — Btu/ft²·hr

for \( 10^3 < GrPr < 10^9 \) \( h = .5 (\Delta T/D'_o)^{1/4} \)

   \( D'_o \) — diameter in inches

vertical plates

\[
10^3 < GrPr < 10^9 \quad h = .28 (\Delta T/L)^{1/4}
\]

   \( L \) in feet