**Problem 1.** This problem is designed to help you think carefully about setting up ODEs. It is a beefier version of Problem 4 on the midterm exam, that brings in additional subtleties. It is a good idea to make sure you know how to set up and solve this kind of problem.

You have two tanks – call them A and B – each of which contains 20 gallons of clean water. You pump salt water with concentration \( c(t) \) into tank A at the rate of 10 gallons/second. You pump salt water with concentration \( d(t) \) into tank B at 10 gallons/second. A pipe carries salt water from tank A to tank B at a rate of 20 gallons/second, and a second pipe carries salt water from B to A at a rate of 10 gallons/second. Finally, a pipe carries the ‘final’ product out of tank B at a rate of 20 gallons/second.

a. Draw a diagram of the situation, clearly labelling the flow rates, volumes, concentrations, etc.

b. Call the number of pounds in tanks A and B \( a(t) \) and \( b(t) \) respectively. Do a mass balance (change in pounds in each tank per unit time) = (pounds in/time) - (pounds out / time) for each tank, and write the corresponding differential equations for \( a(t) \) and \( b(t) \).

c. Laplace Transform these equations, and solve for the Laplace Transformed solutions \( A(s) \) and \( B(s) \) in terms of the Laplace Transformed input concentrations \( C(s) \) and \( D(s) \).

d. The input stream \( c(t) \) is zero until \( t = 4 \) seconds, at which point you switch on the salt to 2 pounds/gallon. The stream \( d(t) \) is zero until \( t = 4 \) seconds, at which point it becomes 3 pounds/gallon, then drops back to zero at \( t = 6 \) seconds. Write expressions for \( c(t) \) and \( d(t) \), and take their Laplace Transforms.

e. Using the input functions from (d), invert the transforms from (c) to obtain the solutions for the amount of salt \( a(t) \) and \( b(t) \) in the tanks as functions of time. What is the concentration of the ‘final product’? (That is, the pipe coming out of tank B at 20 gallons/second)?

**Problem 2.** P. 426, probs. 1, 2, 4, 5, 6, 13

**Problem 3.** P. 432, probs. 1, 2, 4