New Commands:

```mathematica
<<Graphics'Animation'
Animate[Plot[f[x,t],{x,0,1},PlotRange->{0,1}],{t,0,10,.5}]
as = Table[a[n],{n,0,50}]
a20 = as[[20]]
```

Like last week, we’re going to solve the heat equation and animate the solutions.
In class, we solved the heat equation (here I am taking $K$ and $L$ to be 1)

$$\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial x^2}$$  \hspace{1cm} (1)

with non-‘simple’ boundary conditions

$$T(x = 0, t) = 0$$  \hspace{1cm} (2)

$$T(x = 1, t) = 100$$  \hspace{1cm} (3)

and initial conditions

$$T(x, t = 0) = 0.$$  \hspace{1cm} (4)

This involved splitting the solution into a steady bit and a transient bit,

$$T(x, t) = T_s(x) + T_{tr}(x, t).$$  \hspace{1cm} (5)

The steady part

$$T_s(x) = 100x$$  \hspace{1cm} (6)

takes care of the non-simple boundary conditions, and the general solution for the transient part $T_{tr}$ is given by

$$T_{tr}(x, t) = \sum_\beta (a_\beta \cos \beta x + b_\beta \sin \beta x) e^{-\beta^2 t}. $$  \hspace{1cm} (7)

Using the boundary condition at $x = 0$ kills the cosine terms ($a_\beta = 0$) and the boundary condition at $x = 1$ gives the eigenfunction condition –

$$\sin \beta = 0, \text{ or } \beta = n\pi.$$  \hspace{1cm} (8)

This gives a solution

$$T(x, t) = \sum_{n=1}^{\infty} b_n \sin (n\pi x) e^{-n^2\pi^2 t}. $$  \hspace{1cm} (9)

Next, to figure out what the $b_n$’s actually are, we have to use the initial condition for the transient solution, which is the I.C. for the full temperature minus the steady solution:

$$T_{tr}(x, t = 0) = T(x, t = 0) - T_s(x)$$  \hspace{1cm} (10)

$$T_{tr}(x, t) = \sum_{n=1}^{\infty} b_n \sin (n\pi x) = -100x$$  \hspace{1cm} (11)
and zero everywhere else. The problem we now face is written in terms of a Fourier Sine Series, whose coefficients $b_n$ we must find. Recall that these are given by

$$b_n = 2 \int_0^1 (-100x) \sin (n\pi x) \, dx.$$ \hspace{1cm} (12)

**EXERCISES.**

1. (a) Using Mathematica, write a code that will compute the coefficients $b_n$ for the transient solution $T_{tr}(x, t)$.
   (b) Take some reasonable number of terms and plot $T_{tr}(x, t = 0)$ – does the answer look like you think it should?
   (c) Now animate the transient temperature $T_{tr}(x, t)$ – how does it behave as $t$ gets large? Does this make sense?
   (d) Take your individual eigenfunctions for $n = 1, 2, 4, 7$ and animate each – do the eigenfunctions with higher $n$ decay faster or slower than the low-$n$ ones?
   (e) Animate the entire temperature profile $T(x, t) = T_s(x) + T_{tr}(x, t)$. Does it do what it should? (i.e. start with $T = 0$ everywhere, and end with $T(x = 0) = 0$, and $T(x = 1) = 100$)?
   (f) Extra credit: Use COMSOL to solve this same problem. You will want to draw a rectangle of length 1 and whatever height you like. In subdomain/boundary, set the top and bottom walls to be insulating/no-flux, the vertical wall at $x = 0$ to concentration $c = 0$ and the vertical wall on the right to be concentration $c = 100$.

2. Recall, yesterday we solved the 2D steady and transient heat equation for a square plate, whose temperature on the bottom, left, and top sides are all 0, and whose temperature on the right is 100. The steady solution was

$$T_s(x, y) = \sum_{n=0}^{\infty} E_n \sin n\pi y \frac{e^{n\pi x} - e^{-n\pi x}}{e^{n\pi} - e^{-n\pi}}$$ \hspace{1cm} (13)

where

$$E_n = 2 \int_0^1 (100) \sin n\pi y \, dy.$$ \hspace{1cm} (14)

Compute these terms and use either ‘contour plot’ or ‘density plot’

ContourPlot[Ts[x,y,{x,0,1},{y,0,1}]
DensityPlot[Ts[x,y,{x,0,1},{y,0,1}]

to plot out the steady temperature profile on the plate.