Problem 1:

Let \( y(x) \) be the investment resulting from a deposit \( y_o \) and \( x \) years at an annual interest rate \( r \). Show that

\[
y(x) = y_o \left[ 1 + r \right]^x \quad \text{(interest compounded annually)}
\]

\[
y(x) = y_o \left[ 1 + \left( \frac{r}{4} \right) \right]^{4x} \quad \text{(interest compounded quarterly)}
\]

\[
y(x) = y_o \left[ 1 + \left( \frac{r}{365} \right) \right]^{365x} \quad \text{(interest compounded daily)}
\]

Recall from calculus that \( \left[ 1 + \left( \frac{1}{n} \right) \right]^n \to e \) as \( n \to \infty \), hence

\[
\left[ 1 + \left( \frac{r}{n} \right) \right]^n \to e^{rx} \quad \text{which gives}
\]

\[
y(x) = y_o e^{rx} \quad \text{(interest compounded continuously)}
\]

What differential equation does the last function satisfy? Let \( y_o = 1000.00 \) and \( r = 8\% \). Compute \( y(1) \) and \( y(10) \) from each of the four formulas; how much difference is there between daily and continuous compounding?

Problem 2:

Hormone secretion can be modeled by

\[
y' = a - b \cos \frac{2\pi t}{24} - ky
\]

Here, \( t \) is time [in hours, with \( t = 0 \) suitably chosen, e.g., 8:00 A.M.], \( y(t) \) is the amount of a certain hormone in the blood, \( a \) is the average secretion rate, \( b \cos (\pi t/12) \) models the daily 24-h secretion cycle, and \( ky \) models the removal rate of the hormone from the blood. Find the solution when \( a = b = k = 1 \) and \( y(0) = 2 \).
Problem 3:

(3A) Determine the frequency of oscillation of a pendulum of length $L$, as shown in the figure below. Neglect air resistance and the weight of the rod. Assume that $\theta$ is small enough so that $\sin \theta \approx \theta$.

![Figure 3A](image)

(3B) Suppose that the system consists of a pendulum as in (3A) with two springs with constants $k_1$ and $k_2$ attached to the oscillating body and two vertical walls, such that $\theta = 0$ remains the position of static equilibrium and $\theta(t)$ remains small during the motion. Find the period $T$.

![Figure 3B](image)