Dynamic Behavior

In analyzing process dynamic and process control systems, it is important to know how the process responds to changes in the process inputs.

A number of standard types of input changes are widely used for two reasons:

1. They are representative of the types of changes that occur in plants.

2. They are easy to analyze mathematically.
1. Step Input

A sudden change in a process variable can be approximated by a step change of magnitude, $M$:

$$U_s \triangleq \begin{cases} 0 & t < 0 \\ M & t \geq 0 \end{cases}$$

(5-4)

The step change occurs at an arbitrary time denoted as $t = 0$.

- **Special Case**: If $M = 1$, we have a “unit step change”. We give it the symbol, $S(t)$.

- **Example of a step change**: A reactor feedstock is suddenly switched from one supply to another, causing sudden changes in feed concentration, flow, etc.
**Example:**

The heat input to the stirred-tank heating system in Chapter 2 is suddenly changed from 8000 to 10,000 kcal/hr by changing the electrical signal to the heater. Thus,

\[ Q(t) = 8000 + 2000S(t), \quad S(t) \triangleq \text{unit step} \]

and

\[ Q'(t) = Q - \bar{Q} = 2000S(t), \quad \bar{Q} = 8000 \text{ kcal/hr} \]

2. **Ramp Input**

- Industrial processes often experience “drifting disturbances”, that is, relatively slow changes up or down for some period of time.
- The rate of change is approximately constant.
We can approximate a drifting disturbance by a ramp input:

\[
U_R(t) \triangleq \begin{cases} 
0 & t < 0 \\
\text{at} & t \geq 0 
\end{cases} \tag{5-7}
\]

Examples of ramp changes:

1. Ramp a setpoint to a new value. (Why not make a step change?)
2. Feed composition, heat exchanger fouling, catalyst activity, ambient temperature.

3. Rectangular Pulse

It represents a brief, sudden change in a process variable:
\[ U_{RP}(t) \triangleq \begin{cases} 
0 & \text{for } t < 0 \\
h & \text{for } 0 \leq t < t_w \\
0 & \text{for } t \geq t_w 
\end{cases} \]  \hspace{1cm} (5-9)

**Examples:**

1. Reactor feed is shut off for one hour.
2. The fuel gas supply to a furnace is briefly interrupted.
Figure 5.2. Three important examples of deterministic inputs.
4. Sinusoidal Input

Processes are also subject to periodic, or cyclic, disturbances. They can be approximated by a sinusoidal disturbance:

\[ U_{\sin}(t) \triangleq \begin{cases} 
0 & \text{for } t < 0 \\
A \sin(\omega t) & \text{for } t \geq 0 
\end{cases} \quad (5-14) \]

where: \( A = \) amplitude, \( \omega = \) angular frequency

Examples:

1. 24 hour variations in cooling water temperature.
2. 60-Hz electrical noise (in the USA)
5. Impulse Input

- Here, \( U_I(t) = \delta(t) \).
- It represents a short, transient disturbance.

**Examples:**

1. Electrical noise spike in a thermo-couple reading.
2. Injection of a tracer dye.

- Useful for analysis since the response to an impulse input is the inverse of the TF. Thus,

\[
\begin{align*}
U(t) &\xrightarrow{G(s)} y(t) \\
U(s) &\xrightarrow{} Y(s)
\end{align*}
\]

Here,

\[
Y(s) = G(s)U(s) \quad (1)
\]
The corresponding time domain express is:

\[ y(t) = \int_0^t g(t - \tau)u(\tau) d\tau \quad (2) \]

where:

\[ g(t) \triangleq \mathcal{L}^{-1}[G(s)] \quad (3) \]

Suppose \( u(t) = \delta(t) \). Then it can be shown that:

\[ y(t) = g(t) \quad (4) \]

Consequently, \( g(t) \) is called the “impulse response function”.
First-Order System

The standard form for a first-order TF is:

\[ \frac{Y(s)}{U(s)} = \frac{K}{\tau s + 1} \]  \hspace{1cm} (5-16)

where:

- \( K \triangleq \) steady-state gain
- \( \tau \triangleq \) time constant

Consider the response of this system to a step of magnitude, \( M \):

\[ U(t) = M \text{ for } t \geq 0 \quad \Rightarrow U(s) = \frac{M}{s} \]

Substitute into (5-16) and rearrange,

\[ Y(s) = \frac{KM}{s(\tau s + 1)} \]  \hspace{1cm} (5-17)
Take $\mathcal{L}^{-1}$ (cf. Table 3.1),

$$y(t) = KM \left(1 - e^{-t/\tau}\right)$$  \hspace{1cm} (5-18)

Let $y_\infty \triangleq$ steady-state value of $y(t)$. From (5-18), $y_\infty = KM$.

\begin{center}
\begin{tabular}{c|c}
$t$ & $y$ \\
\hline
0 & 0 \\
$\tau$ & 0.632 \\
$2\tau$ & 0.865 \\
$3\tau$ & 0.950 \\
$4\tau$ & 0.982 \\
$5\tau$ & 0.993 \\
\end{tabular}
\end{center}

*Note:* Large $\tau$ means a slow response.
Integrating Process

Not all processes have a steady-state gain. For example, an “integrating process” or “integrator” has the transfer function:

\[ \frac{Y(s)}{U(s)} = \frac{K}{s} \quad (K = \text{constant}) \]

Consider a step change of magnitude \( M \). Then \( U(s) = M/s \) and,

\[ Y(s) = \frac{KM}{s^2} \overset{\mathcal{L}^{-1}}{\Rightarrow} y(t) = KMt \]

Thus, \( y(t) \) is unbounded and a new steady-state value does \textit{not} exist.
Common Physical Example:

Consider a liquid storage tank with a pump on the exit line:

- Assume:
  1. Constant cross-sectional area, $A$.
  2. $q \neq f(h)$
- Mass balance: $A \frac{dh}{dt} = q_i - q \ (1) \Rightarrow 0 = \bar{q}_i - \bar{q} \ (2)$
- Eq. (1) – Eq. (2), take $\mathcal{L}$, assume steady state initially, $H'(s) = \frac{1}{As} \left[ Q'_i(s) - Q'(s) \right]$ 
- For $Q'(s) = 0$ (constant $q$), $\frac{H'(s)}{Q'_i(s)} = \frac{1}{As}$
Second-Order Systems

• Standard form:

\[
\frac{Y(s)}{U(s)} = \frac{K}{\tau^2 s^2 + 2\zeta \tau s + 1}
\]  

which has three model parameters:

\[
\begin{align*}
K & \triangleq \text{steady-state gain} \\
\tau & \triangleq \text{"time constant"} \ [= \text{time}] \\
\zeta & \triangleq \text{damping coefficient (dimensionless)}
\end{align*}
\]

• Equivalent form: \( \left( \omega_n \triangleq \text{natural frequency} = \frac{1}{\tau} \right) \)

\[
\frac{Y(s)}{U(s)} = \frac{K \omega_n^2}{s^2 + 2\zeta \omega_n s + \omega_n^2}
\]
The type of behavior that occurs depends on the numerical value of damping coefficient, $\zeta$:

It is convenient to consider three types of behavior:

<table>
<thead>
<tr>
<th>Damping Coefficient</th>
<th>Type of Response</th>
<th>Roots of Character. Polynomial</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\zeta &gt; 1$</td>
<td>Overdamped</td>
<td>Real and $\neq$</td>
</tr>
<tr>
<td>$\zeta = 1$</td>
<td>Critically damped</td>
<td>Real and $=$</td>
</tr>
<tr>
<td>$0 \leq \zeta &lt; 1$</td>
<td>Underdamped</td>
<td>Complex conjugates</td>
</tr>
</tbody>
</table>

Note: The characteristic polynomial is the denominator of the transfer function:

$$\tau^2 s^2 + 2\zeta \tau s + 1$$

What about $\zeta < 0$? It results in an unstable system.
Figure 5.8. Step response of underdamped second-order processes.
Figure 5.9. Step response of critically-damped and overdamped second-order processes.
Several general remarks can be made concerning the responses shown in Figs. 5.8 and 5.9:

1. Responses exhibiting oscillation and overshoot \((y/KM > 1)\) are obtained only for values of \(\zeta\) less than one.

2. Large values of \(\zeta\) yield a sluggish (slow) response.

3. The fastest response without overshoot is obtained for the critically damped case \((\zeta = 1)\).
Figure 5.10. Performance characteristics for the step response of an underdamped process.
1. **Rise Time:** $t_r$ is the time the process output takes to first reach the new steady-state value.

2. **Time to First Peak:** $t_p$ is the time required for the output to reach its first maximum value.

3. **Settling Time:** $t_s$ is defined as the time required for the process output to reach and remain inside a band whose width is equal to $\pm5\%$ of the total change in $y$. The term 95% response time sometimes is used to refer to this case. Also, values of $\pm1\%$ sometimes are used.

4. **Overshoot:** OS = $a/b$ (% overshoot is $100a/b$).

5. **Decay Ratio:** DR = $c/a$ (where $c$ is the height of the second peak).

6. **Period of Oscillation:** $P$ is the time between two successive peaks or two successive valleys of the response.