1. Consider the radioactive decay of $^{239}\text{U} \rightarrow ^{239}\text{Np} \rightarrow ^{239}\text{Pu}$. We can model these three reactions by the ODEs

$$\frac{d[U]}{dt} = -k_u[U]$$
$$\frac{d[Np]}{dt} = k_u[U] - k_n[Np]$$
$$\frac{d[Pu]}{dt} = k_n[Np]$$

(a) Solve these ODEs using Mathematica by the following method. First solve Eq. (1) to obtain an expression for the time dependent concentration $[U](t)$. Assume that 100% of the starting material is uranium (U). Then, substitute your solution for $[U](t)$ into Eq. (2) and solve for $[Np](t)$. Finally, use these results to obtain an expression for $[Pu](t)$. Hint: $[U](t) + [Np](t) + [Pu](t) = 1$

(b) The half-life for the first decay $^{239}\text{U} \rightarrow ^{239}\text{Np}$ is 23.5 minutes and for the second decay $^{239}\text{Np} \rightarrow ^{239}\text{Pu}$ is 2.35 days. Plot your solutions from part (a) as a function of time using these half-lives. Note: The half-life $\tau_{1/2}$ of a substance is the time required for half the substance to decay. Therefore, it is related to the rate constant $k$ by the equation

$$e^{-k\tau_{1/2}} = 1/2$$

2. Use the Table command of Mathematica to build a list containing the function $f(x) = x^2e^{-x}$ evaluated at the points $x = \frac{1}{2}, 1, \frac{3}{2}, 2, \frac{5}{2}, ..., 5$. Build an interpolating function using different values for InterpolationOrder, say 1, 2, and 3. Plot these, along with the list. Which choice gives the best approximation? Which choices give poorer approximations? Why?
3. Make up a $4 \times 4$ matrix of your own choosing. Devise a sequence of *Mathematica* commands that extracts the third column of the matrix and saves it as a four-element list. You may want to load the package `<< Calculus`VectorAnalysis`.

4. a) Use the *Table* command to build a $5 \times 5$ matrix in *Mathematica* where the $(i, j)$ element is given by $2\sin(i\pi/2)\cos(j\pi/3)$.
   
   (b) Repeat part (a) but use an *Array* command to declare the array and the a *Do* loop to fill it with the appropriate elements.

5. For two 3-component vectors $\textbf{A}$ and $\textbf{B}$, explain what the differences (if any) between the results you get in *Mathematica* when you execute the commands “$\textbf{A} \cdot \textbf{B}$”, “$\textbf{A} \times \textbf{B}$”, and “*CrossProduct*[\textbf{A}, \textbf{B}]”. Illustrate the differences with an example.