1. Consider the following set of linear algebraic equations:

\[
\begin{align*}
3x_1 + 4x_2 + x_3 &= 3 \\
-x_1 + x_2 + 2x_3 &= 4 \\
2x_1 - x_2 - x_3 &= -1 .
\end{align*}
\]

(a) Row-reduce this set of equations following the Gauss elimination procedure to find the solution. You may use Mathematica only to multiply and/or add rows together. This gives you practice on how to address individual rows in a matrix.

(b) Use Demo 9 posted on our class web site to solve the problem and check your results.

(c) Use the RowReduce function in Mathematica to check it yet again.

2. (a) Use Mathematica to multiply two nontrivial $6 \times 6$ matrices of your choice together. What are the determinants of your two matrices (use Mathematica)?

(b) Find the inverse of each of these matrices. Show by explicit calculation for your example that $(AB)^{-1} = B^{-1}A^{-1}$. For your example, do you find that $(AB)^{-1} \neq A^{-1}B^{-1}$? Is this result generally valid? Show that $\det A^{-1} = 1/\det A$.

3. The exponential of a square matrix $A$ can be defined by the power series:

\[
e^A = \sum_{n=0}^{\infty} \frac{1}{n!} A^n
\]

where $I$ is the unit (identity) matrix.

(a) Use a Do loop to print out the results of the sum in Eq. (2) term by term up to $n = 10$ for the matrix:

\[
A = \begin{pmatrix}
-1 & -1 \\
1 & -1
\end{pmatrix}.
\]

Hint: Use the Mathematica function MatrixPower[A,n] to evaluate $A^n$. Note that writing $A^n$ does not in general give the same result as MatrixPower[A,n].
(b) Use the function `MatrixExp` to obtain a symbolic expression for \( \exp(A) \). Evaluate this expression numerically and compare it to what you obtained using the Taylor series expression for \( n = 10 \). Within *Mathematica*, does `MatrixExp[A]` give the same result as `Exp[A]`? If not, how do the two differ?

(c) Use the function `MatrixExp` to obtain a symbolic expression for \( e^B \) and \( e^C \) where

\[
B = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad (4)
\]

and

\[
C = \begin{pmatrix} 0 & -x \\ x & 0 \end{pmatrix} \quad (5)
\]

Ponder your results.