1. Consider the matrix
\[
A = \begin{pmatrix}
1 & 2 & 1 & 4 \\
3 & 4 & 0 & 1 \\
2 & 1 & -2 & 3 \\
2 & 3 & 1 & 1
\end{pmatrix}.
\] (1)

(a) Use Mathematica to find the eigenvalues and eigenvectors of \( A \). Hints: Make your life easier by expressing the eigenvalues \( \{\lambda_i\} \) and eigenvectors \( \{e_i\} \) in numerical (decimal) form. Recall that you can extract pieces of vectors and matrices in Mathematica using commands like \( A[[i]] \) and \( A[[i,j]] \).

(b) As a check on your work in part (a), find the eigenvalues of \( A \) by finding the roots of the equation \( \det(A - \lambda I) = 0 \).

(c) Show by explicit calculation of \( A \cdot e_i \) and \( \lambda_i e_i \) that each of the eigenvectors and eigenvalues \( e_i \) and \( \lambda_i \) of \( A \) satisfy the equation
\[
A \cdot e_i = \lambda_i e_i.
\] (2)

(d) Use Mathematica to show that the four eigenvectors are linearly independent and therefore form a basis which spans \( R^4 \). Useful point: The Chop command in Mathematica replaces values smaller than \( 10^{-10} \) with 0 and can be useful for obtaining cleaner output when doing numerical work in Mathematica.

(e) Find a \( 4 \times 4 \) matrix \( B \) such that \( B^{-1}AB \) is diagonal. Demonstrate that \( B^{-1}AB \) is diagonal by explicit calculation using Mathematica. Note that the diagonal elements of \( B^{-1}AB \) are the eigenvalues of \( A \).