1. Consider the system of coupled differential equations described by the matrix equation:

\[
\frac{d\mathbf{x}}{dt} = A\mathbf{x}.
\]  

For each of the coefficient matrices \(A\) listed below, determine whether the origin for the matrix equation above is a sink, source, saddle, or center. Plot the direction field for each using \(\text{Mathematica}\). In cases where the solution consists of real eigenvectors, show the eigendirections on your direction field plot.

(a) \(A = \begin{pmatrix} 0 & 3 \\ 6 & 0 \end{pmatrix}\)
(b) \(A = \begin{pmatrix} -2 & 1 \\ 1 & 2 \end{pmatrix}\)
(c) \(A = \begin{pmatrix} 0 & \frac{1}{2} \\ -2 & 2 \end{pmatrix}\)
(d) \(A = \begin{pmatrix} -6 & 1 \\ -9 & -6 \end{pmatrix}\)
(e) \(A = \begin{pmatrix} -1 & 1 \\ -5 & 1 \end{pmatrix}\)
(f) \(A = \begin{pmatrix} 2 & \frac{1}{3} \\ -3 & -2 \end{pmatrix}\)
(g) \(A = \begin{pmatrix} -1 & 1 \\ -5 & 1 \end{pmatrix}\)
(h) \(A = \begin{pmatrix} -2 & 4 \\ -1 & 0 \end{pmatrix}\)

2. Consider the coupled linear first order linear differential equations

\[
x'(t) = -\frac{1}{4}x(t) + 2y(t)
\]
\[
y'(t) = -2x(t) - \frac{1}{4}y(t),
\]

subject to the initial conditions

\[
x(0) = 0
\]
\[
y(0) = 1.
\]

(a) Use the \texttt{DSolve} function of \textit{Mathematica} to find the solutions for \(x(t)\) and \(y(t)\). Then use \textit{Mathematica} to plot these solutions as a function of \(t\).
(b) Use the `ParametricPlot` function of Mathematica to make a graph of the $x$-$y$ the solution trajectory. You should obtain a spiral solution. Next, use Mathematica to generate a direction (or slope) field plot of $x$ vs. $y$ (recall how you did something very similar to this in Homework #2). Finally, overlay the two plots so that you can see how the trajectory threads the direction field plot. Describe in words how the trajectory would change if we have different initial conditions from those given above. Do you expect all trajectories to have the same end point?

(c) Rewrite equations (1) and (2) as a single equation in matrix form. Then use Mathematica to find the eigenvalues and eigenvectors of the coefficient matrix that appears in your equation. What do your results tell you about the nature of the solutions to equations (1) and (2)? Are they consistent with the particular solutions you found in part (a)?