1. In class and in Demo 12 we analyzed solutions to the nonlinear ODE

\[ x'' + x' - x + x^3 = f_0 \cos t , \]  

for different values of the amplitude \( f_0 \) of the forcing term.

(a) In this part, we look at the effect of changing the power of the nonlinear term. That is, we wish to look at solutions to the ODE

\[ x'' + x' - x + x^n = f_0 \cos t , \]  

for different values of \( n \). Following the methods used in Demo 12, find the critical values (to two significant digits) of the forcing parameter \( f_0 \) where period doubling occurs and where the solutions appear to become chaotic for \( n = 2, 4, 5 \) (\( n = 3 \) is done in the demo). [Hints: These events occur in all the above cases for \( 0 < f_0 < 1 \). Use parametric \( x-y \) plots to find where period doubling transitions occur.] What happens to the solutions when \( n = 1 \)? Briefly explain.

(b) The case of \( n = 3 \) is particularly interesting. Repeat part (a) for this case and show that period doubling first occurs near \( f_0 = 0.69 \) and that the solution appears to become chaotic near \( f_0 = 0.80 \). Be careful to make sure that the initial transients have died out; this can be a little tricky as the value of \( f_0 \) is increased. Investigate what happens as \( f_0 \) is increased from 0.80 to 0.90 and show that the \( x-y \) orbits become less complex at some point. What happens as \( f_0 \) is increase above 0.9? Do the solutions go through another period doubling sequence?